

—Supporting Information—

Novel Solar Cell Materials: Insights From First Principles

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Overview of theoretical methods

The studies summarized in the manuscript utilize a variety of theories, a brief overview of which is given below. A detailed discussion on appropriate approximations to invoke in solid-state QM calculations can be found in the reviews by Bendavid and Carter,¹ Cramer and Truhlar,² and Harvey.³ Examples of how QM calculations can be used to describe TMOs and their alloys for allied phenomena such as photocatalysis can be found in the review by Liao and Carter,⁴ and Le Bahers *et al.*⁵

Bulk energies

A variety of approximations within the general framework of the density functional theory (DFT)^{6,7} can be used for quantitative or qualitative evaluation of energetic and electronic properties, including the systems of interest in this review, *i.e.*, pure, doped, or alloyed NiO, CoO, FeO, Cu₂O, and Cu₂ZnSnS₄ (CZTS).^{8–12} The most important term that dictates the accuracy of DFT calculations is the choice of the necessarily approximate exchange-correlation (XC) functional.¹³ Common XC functionals for condensed matter include the local density approximation (LDA)^{14,15} and the Perdew-Burke-Ernzerhof (PBE) implementation of the generalized gradient approximation (GGA).¹⁶ The strongly constrained appropriately normed

(SCAN) functional¹⁷ introduced recently was shown to satisfy all 17 known constraints for the behavior of an XC functional, unlike the GGA or the LDA,¹³ a truly impressive achievement. Gautam *et al.*¹² therefore employed SCAN to understand the influence of dopants on defect energetics in CZTS. Although SCAN is formally very appealing, density of states (DOS) calculations in CZTS¹² reveal that SCAN still exhibits the typical failure of pure density functionals: it fails to reproduce experimental band gaps. The lack of a derivative discontinuity in all pure density XC functionals is the source of this error.¹⁸

Pure DFT approximations, especially the LDA and the GGA, suffer from self-interaction errors (SIE),¹⁵ owing to an inaccurate description of electron exchange interactions. Consequently, such approximations often fail to reproduce ground-state properties such as formation energies and lattice parameters,^{19–24} especially in open-shell, transition-metal compounds.²⁵ Additionally, as a ground-state theory, pure DFT approximations usually underestimate band gaps of semiconductors.^{26,27} Prior studies have shown that either adding a Hubbard U correction to the DFT Hamiltonian (resulting in a DFT+ U framework)^{28,29} or incorporating some exact exchange from Hartree-Fock (HF) theory via hybrid functionals (*e.g.*, Heyd-Scuseria-Ernzerhof, HSE³⁰ or PBE0³¹) often yield better quantitative estimations of formation energies and qualitative predictions of band gaps. Specifically, DFT+ U has the advantage of not adding significant computational cost to a DFT calculation, unlike hybrid functionals in periodic planewave basis calculations that can be two orders of magnitude more expensive.³⁰ However, the magnitude of U is not known *a priori* and must be separately determined and validated, preferably by *ab initio* approaches.^{32–35} Hence, the DFT+ U framework, among others, was used to study the bulk stability of all the TMO systems reviewed in the main text (NiO, CoO, FeO, and Cu₂O). The specific U values employed on the transition metal centers are also indicated within the appropriate sections

of the main text. Interestingly, based on benchmarking SCAN and experimental formation energies of binary sulfides, Gautam *et al.*¹² concluded that a + U correction with SCAN was not required to accurately describe the ground-state properties of CZTS.

Band gaps

For quantitative comparisons of calculated band gaps with experiments, a ground-state theory, such as DFT, DFT+ U , hybrid DFT (e.g., HSE), is not of much utility since the unoccupied virtual bands (*i.e.*, conduction bands) are not described accurately. Thus, advanced excited-state methods, such as the many-body Green's function theory (GW),^{36,37} are often employed for this purpose. Notably, GW theory provides a framework to calculate the self-energy of a many-electron system via an expansion of the single-particle Green's function (G) that describes the propagation of an electron within a screened Coulombic potential (W). Physically, the GW approximation is equivalent to adding a “dynamically screened” Coulomb potential³⁸ to DFT.

Although self-consistent GW calculations can be performed, prior studies have shown that doing a non-self-consistent GW calculation (G_0W_0)³⁹ as a perturbation to the ground-state one-electron wavefunctions and eigenvalues obtained from DFT (*i.e.*, DFT+ G_0W_0) often yields accurate band gaps even in small-gap semiconductors.⁴⁰⁻⁴³ G_0W_0 calculates a quasiparticle (QP) band gap directly comparable to a measured photoemission/inverse photoemission gap. The accuracy of G_0W_0 calculations, however, relies on the quality of the ground-state wavefunctions obtained from DFT and the use of a sufficient number of virtual states to obtain a reliable band gap.⁴⁴ Since DFT+ U provides a better description of ground-state wavefunctions in open-shell systems, DFT+ U + G_0W_0 calculations can furnish accurate predictions of transition-metal oxide band gaps.^{45,46} Thus, based on benchmarking theoretical predictions with experiments, Alidoust and Carter^{10,47} used PBE+ U + G_0W_0 and LDA+ U + G_0W_0 for band-gap calculations in (pure and

doped) NiO and CoO, respectively, where the magnitude of U values used for Ni and Co were obtained from size-converged electrostatically embedded unrestricted HF calculations.^{48,49}

Charge transfer barriers

Charge transport in transition metal oxides is most often best described by using the small polaron model.^{50,51} A small polaron is the combination of a localized carrier (hole or electron) and a localized lattice distortion that affects a few surrounding ions (distances less than the lattice constant) from the localized carrier.⁵⁰⁻⁵⁴ In the case of (pure, doped, and alloyed) NiO and FeO, as modeled by Alidoust and Carter,^{55,56} and Toroker and Carter,⁵⁷ respectively, a small hole polaron is predicted to form when a hole localizes on an oxygen in NiO (*i.e.*, forming O^{1-} from O^{2-}) and *via* hole localization on an iron in FeO (*i.e.*, forming the stable, half-filled d -shell Fe^{3+} from Fe^{2+}). These are the most stable positions of the hole, as opposed to forming Ni^{3+} or O^{-1} in NiO and FeO, respectively. Hole polarons can migrate within a lattice from a donor (O^{1-} in NiO and Fe^{3+} in FeO) to an acceptor site (O^{2-} in NiO, Fe^{2+} in FeO), limited by a barrier ΔG^* , which can be estimated using the Marcus theory of electron transfer (*vide infra*).^{58,59}

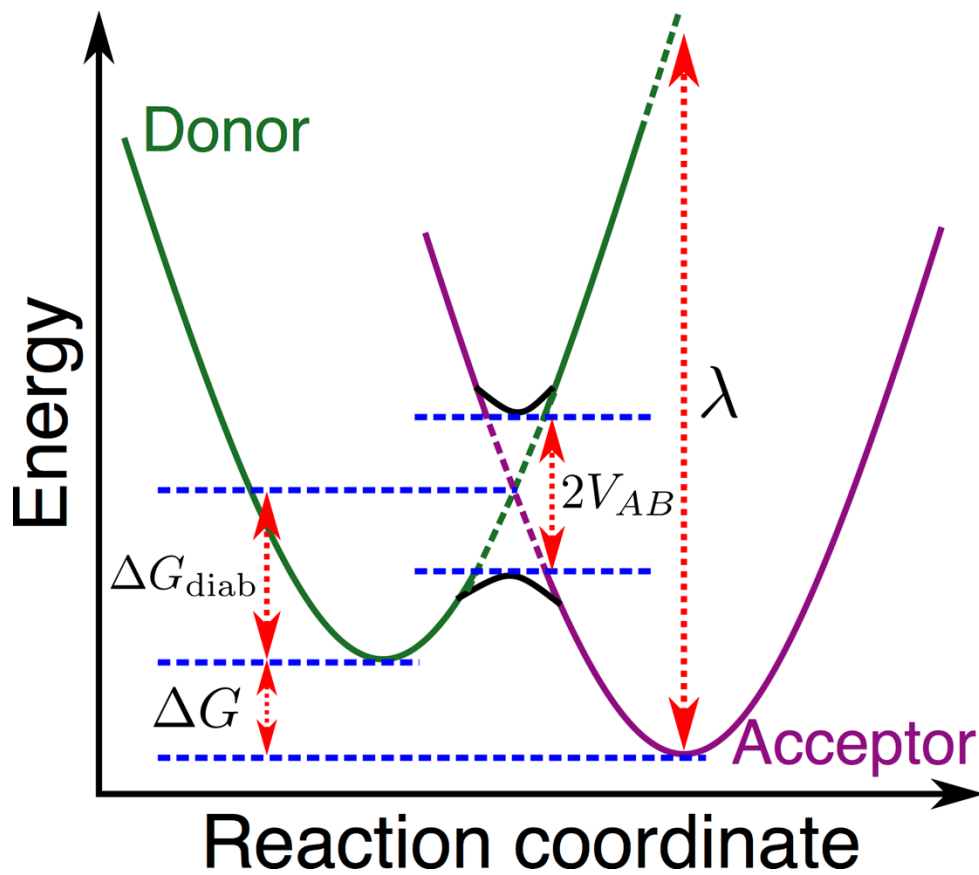


Figure S1: Schematic of potential energy surfaces during CT within the framework of Marcus theory.⁵⁸

Figure S1 displays a schematic of a parabolic potential energy surface (within the framework of Marcus theory) around a donor (D, green curve) site and an acceptor (A, purple) site during charge (*e.g.*, hole polaron) transfer in a material. Note that the donor and acceptor states are defined by a set of collective nuclear coordinates, q_D and q_A , respectively, and the free energy difference between the two states is given by ΔG in **Figure S1**. If the potential energy surfaces are known for the donor and acceptor states, then one can calculate a reorganization energy (λ in **Figure S1**) as the energy required to distort the lattice from the equilibrium configuration of the donor ($q_{D, \text{equil}}$) to the equilibrium configuration of the acceptor ($q_{A, \text{equil}}$) without the hole (or electron) being transferred. The actual CT process can occur under two distinct scenarios:⁵⁸ *i*) the donor and acceptor nuclei do not move when the charge is transferred, *i.e.*, nuclear and charge

motion are decoupled, referred to as the “diabatic” approximation; and *ii*) both the donor and acceptor move during CT, *i.e.*, the nuclei and charge motion are highly coupled, referred to as the “adiabatic” approximation.

In the diabatic CT scenario (dashed and solid green and purple lines in **Figure S1**), the transferring charge has to “jump” from the donor energy surface to the acceptor energy surface at the crossing point (q_c), *i.e.*, the point of intersection between the green and purple dashed lines in **Figure S1**. In the case of adiabatic CT (solid black curves), the potential energy curves split into distinct ground- and excited-state configurations at q_c , where the extent of coupling between the donor and acceptor energy curves is quantified via the coupling matrix element, V_{AB} . Note that the energy difference between the ground- and excited-state configurations at q_c (ΔE) is twice that of the coupling matrix element, *i.e.*, $\Delta E = 2V_{AB}$.

In general, ΔG^* for CT from the donor to the acceptor ($D \rightarrow A$), given λ and V_{AB} , can be expressed as $\Delta G^* = -\frac{\lambda}{4} + \frac{(\lambda^2 + 4V_{AB}^2)^{\frac{1}{2}}}{2} - V_{AB}$,⁶⁰ where the CT barrier for $A \rightarrow D$ is $\Delta G^* + \Delta G$. Under the diabatic approximation, $V_{AB} \approx 0$, and $\Delta G^* = \Delta G_{diab} \sim \frac{\lambda}{4}$, which signifies that the barrier can be efficiently calculated if the ground-state energy curves of the donor and acceptor states are known (computed via robust, self-consistent, ground-state methods). In contrast, under the adiabatic approximation ($0 < V_{AB} \ll \frac{\lambda}{2}$), ΔG^* reduces from ΔG_{diab} by approximately the coupling element, *i.e.*, $\Delta G^* = \Delta G_{adiab} \sim \frac{\lambda}{4} - V_{AB}$,⁶¹ necessitating the explicit evaluation of both the ground- and excited-state energies at q_c . It is difficult to determine *a priori* if ΔG_{adiab} is substantially different from ΔG_{diab} for a given material. Hence, performing a few representative calculations of V_{AB} to verify the (a)diabatic nature of CT is recommended.⁵⁵

Following previous theoretical work^{54,62,63} calculating polaron migration barriers in pure and doped TMOs, Alidoust and Carter^{55,56} and Toroker and Carter⁵⁷ estimated the diabatic barriers (ΔG_{diab}^*) in NiO and FeO, respectively, using structurally relaxed unrestricted Hartree-Fock (UHF⁶⁴) calculations on electrostatically embedded clusters. Carved out of rocksalt NiO (FeO) periodic crystals, the clusters were capped with Mg effective core potentials (ECPs)⁶⁵ to compensate for missing Ni²⁺ (Fe²⁺) ions adjacent to O²⁻ and to account for Pauli repulsion between cluster electrons and neighboring core electrons of the +2 ions. The ECP-capped cluster was then embedded within an aperiodic point charge array to represent the electrostatic potential arising from the surrounding crystal structure.⁶⁶ The intermediate states were obtained via the linear coordinate approximation after optimizing the geometry of the donor and acceptor hole configurations.⁶⁷

To calculate the adiabatic hole transfer barrier in NiO, Alidoust and Carter^{55,56} employed complete active space self-consistent field (CASSCF) calculations⁶⁸ to evaluate the excited-state energy. Note that a single-determinant, ground-state theory, such as UHF, cannot accurately describe the coupled wavefunctions (and the consequent split in energy levels) at the transition state. The computationally expensive CASSCF calculations instead includes all possible excited state configurations within a subspace (called the active space) in its wavefunction, along with a self-consistent optimization of the orbital shapes that give rise to those configurations.

Lattice disorder

Lattice disorder, or any order-disorder phase transformation in a given solid, can be described via a cluster expansion (CE⁶⁹) model. A CE is a generalized Ising model⁷⁰ that describes the total energy of a given lattice in terms of atomic occupations over various lattice sites. In practice, CE

models are often written as a truncated series of cluster interactions, such as interactions between pairs, triplets, *etc.*⁷¹ The truncated series then is fit to an input set of configurations (typically built from the supercells of a given structure) and their corresponding energies (calculated *via* DFT or a similar theory). In combination with canonical Monte Carlo (MC) simulations, where the CE evaluates the energy during each MC sweep, CE models can be used to estimate the configurational entropy of a lattice and have been shown to accurately estimate order-disorder transitions.⁷²⁻⁷⁴

To investigate the origins of disorder within the CZTS structure (in other words, to probe the order-disorder transition within CZTS), Yu and Carter^{11,75} performed MC simulations based on a CE model built on a set of PBE+*U* calculations in CZTS. The *U* values of Cu, Zn, and Sn used in the PBE+*U* calculations were determined from previous electrostatically embedded unrestricted HF calculations.³⁵ To construct the CE, the authors⁷⁵ considered four different pair and triplet interactions. Note that Yu and Carter^{11,75,76} also added long-range dispersion corrections, namely Grimme's *a posteriori* *D2* correction,⁷⁷ in all of their PBE+*U* calculations (resulting in a PBE+*U*+*D2* functional) to improve predictions of the lattice constants and the bulk modulus.⁷⁶

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