

# Materials Design Rules for Multi-Valent Ion Mobility in Intercalation Structures –Supporting Information–

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## 1.1 Computational Details

To model the ionic diffusion in the selected host structures, we use nudged elastic band (NEB)<sup>11</sup> simulations coupled with density functional theory (DFT), specifically the Perdew–Burke–Ernzerhof<sup>2</sup> Generalized-Gradient Approximation (GGA), as implemented in VASP. We elected to use GGA over GGA+ $U$  for the reasons previously indicated by Liu *et al.*<sup>3</sup> for first-principles calculations of MV ion migration in oxide spinels, namely the pronounced metastability of electronic states along the ion migration path with + $U$  which results in poor computational convergence, and also the lack of conclusive evidence that + $U$  performs better in determining the cation migration barriers.

The total energy was sampled using a Monkhorst-Pack mesh with  $k$ -point density of 1000/(number of atoms per unit cell). Projector augmented-wave theory<sup>4</sup> combined with a well-converged plane-wave cutoff of 520 eV were used to describe the wave functions. The convergence threshold of the total energy was set to  $1 \times 10^{-5}$  eV, and a tolerance of 0.1 eV/Å for the forces was used to converge the minimum energy path (MEP) in the NEB procedure. The initial MEPs were prepared by linear interpolation

between the end-points using 8 images (or 9 for  $V_2O_5$ ). In order to avoid spurious interactions between images, supercells of appropriate sizes were used ensuring a minimum inter-image distance of 8 Å.

With the computational parameters described above, we performed the NEB calculations on several host-structure while changing the diffusing species of interest. The host-structures considered are  $FePO_4$  (olivine),  $Mn_2O_4$  (spinel),  $NiO_2$  (layered),  $V_2O_5$  (pseudo-layered), while the moving species chosen were  $Li^+$ ,  $Ca^{2+}$ ,  $Mg^{2+}$ ,  $Zn^{2+}$ , and  $Al^{3+}$ . The analysis of the MEP not only gives the topology of the diffusing species, but also provides information about the host-structure perturbation. The maximum energy value of the MEP represents the migration barrier that is reported in this work.

For the calculation of voltages and capacities (see Sec. 1.5) DFT+ $U$  (PBE+ $U$ ) was used to correct for the self-interaction error introduced by the highly localized  $d$  orbitals of the redox species V, Mn, Fe, and Ni. The  $U$  values were set to 3.25 eV for V, 3.1 eV for Mn, 5.3 eV for Fe and 6.45 eV for Ni, respectively.

## 1.2 $Al^{3+}$ Calculations in Olivine $FePO_4$ and Layered $NiO_2$

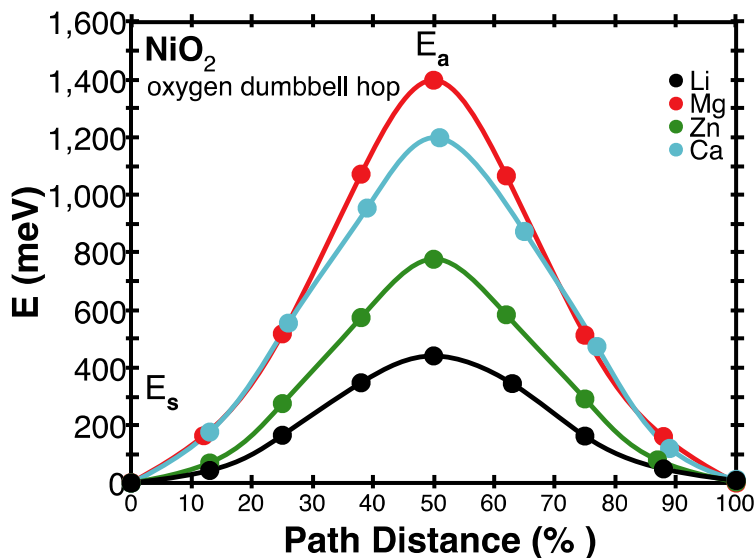
NEB calculations for  $Al^{3+}$  are particularly challenging to converge in olivine  $FePO_4$  and layered  $NiO_2$  due to the highly localized charge on the  $Al^{3+}$  species and the strong thermodynamic instability of  $Al^{3+}$  in these two host structures resulting in especially large forces on the host lattice (which consequently make convergence difficult). For this reason we presented only the MEP of  $Al^{3+}$  in  $Mn_2O_4$  and  $V_2O_5$ . In lieu of NEB

results for olivine  $\text{FePO}_4$  and layered  $\text{NiO}_2$ , we have calculated the site energy difference  $E_i - E_s$  between  $\text{Al}^{3+}$  residing in the initial octahedral site and the intermediate tetrahedral site along the diffusion path. For both of these cases, we find  $\text{Al}^{3+}$  occupation of the intermediate tetrahedral site is indeed lower in energy (by  $\sim 136$  meV for olivine  $\text{FePO}_4$ , and  $\sim 226$  meV for layered  $\text{NiO}_2$ ), verifying the thermodynamic instability of  $\text{Al}^{3+}$  in these structures.

### 1.3 Estimation criteria for $E_m$ cutoff

An upper bound for the MV migration barrier  $E_m$  can be established from reasonable battery performance criteria: a 2 hour (dis)charge time  $t$  (or C/2 rate) for a 1  $\mu\text{m}$  active particle size suggests a minimum diffusivity  $D \sim 10^{-12} \text{ cm}^2\text{s}^{-1}$  given the diffusion length scales as  $\sqrt{Dt}$ . Using a random walk for diffusion sets a maximum  $E_m \sim 525$  meV that can be tolerated, assuming  $D \approx v \cdot a^2 \cdot \exp(-E_m/kT)$  with atomic jump frequency  $v \approx 10^{12} \text{ s}^{-1}$  and atomic jump distance  $a \approx 3 \text{ \AA}$ , the length of a typical lattice parameter. For every order of magnitude particle size reduction this tolerance increases by  $\sim 125$  meV. Hence, 100 nm crystallites could be charged and discharged in 2 hours when barriers are less than  $\sim 650$  meV. Note that reasonable ion diffusion is a required condition for cathode materials, but it is by no means sufficient as other phenomena, either in the cathode (e.g. phase transformations, conductivity) or in the cell, can be rate limiting. Nonetheless, solid-state diffusion is widely seen as the most challenging design problem for MV-cathode materials.

### 1.4 $\text{NiO}_2$ Oxygen Dumbbell Hop



**Figure S1:** Energies for MV ion migration ( $\text{Li}^+$ ,  $\text{Mg}^{2+}$ ,  $\text{Zn}^{2+}$ ,  $\text{Ca}^{2+}$ ) for the oxygen dumbbell hop in layered  $\text{NiO}_2$ , which correspond to MV ion motion through a O–O bond (labeled as  $E_a$ ), are higher for each ion than migration through the intermediate tetrahedral site shown in **Figure 1c**.

### 1.5 Comparison to First-Principles data for Orthorhombic $\text{V}_2\text{O}_5$ in the literature

Carrasco *et al.*<sup>5</sup> and Zhou *et al.*<sup>6</sup> both have investigated  $\text{Mg}^{2+}$  intercalation in orthorhombic  $\text{V}_2\text{O}_5$  polymorphs using first-principles NEB calculations, but with some deviation from our outlined computational procedure. Carrasco *et al.* investigated  $\text{Li}^+$ ,  $\text{Mg}^{2+}$ , and  $\text{Ca}^{2+}$  migration in the de-intercalated  $\alpha$ - $\text{V}_2\text{O}_5$  polymorph (note in this work we have focused on the  $\delta$  polymorph), incorporating both  $+U$  and van der Waal’s interactions in their calculations, and obtained migration energies of  $\sim 310$ ,  $\sim 980$  and  $\sim 1800$  meV for  $\text{Li}^+$ ,  $\text{Mg}^{2+}$ , and  $\text{Ca}^{2+}$ , respectively. Using the parameters outlined in the **Computational Details** section, we performed calculations in the same structure and obtained very good agreement for  $\text{Mg}^{2+}$  (975 meV) and  $\text{Ca}^{2+}$  (1700 meV), but a lower value for  $\text{Li}^+$  (150 meV) as also noted in Carrasco’s work.

Zhou *et al.* investigated  $\text{Mg}^{2+}$  diffusion in both the deintercalated  $\alpha$  and  $\delta$  polymorphs, and obtain a much higher value for  $\alpha$ ,  $>1200$  meV compared to  $\sim 975 - 980$  meV as indicated by our calculations and calculations performed by Carrasco. For the  $\delta$  polymorph, the shape of the migration energy has the same “valley” shape as seen in **Fig 3** in this work, but both the migration barrier and intermediate site energy determined by Zhou *et al.* are much higher,  $>1800$  meV compared to 680 meV for the migration barrier, and 800 meV compared to less than 200 meV for the intermediate site energy. We speculate that the differences originate from two discrepancies in computational procedure. First, for some calculations Zhou *et al.* allow only the Mg ions and nearby oxygen ions to move which impedes convergence to the MEP, and second, we found that using an odd number of images (9 as opposed to 8) has a significant effect on improving convergence to the MEP by allowing the middle image to occupy the low-energy intermediate site, which is observed for all MV cations considered in both the charged and discharged limits as observed in **Fig 3**.

## 1.6 Voltages and Capacity of multivalent cathodes

Table S1 shows the voltages and specific capacities of the materials discussed in the paper calculated from first-principle calculations (see Sec. 1.1).

**Table S1:** Voltages and Specific Capacities of  $\text{M}_x\text{NiO}_2$ ,  $\text{M}_x\text{FePO}_4$ ,  $\text{M}_x\text{Mn}_2\text{O}_4$ ,  $\delta\text{-M}_x\text{V}_2\text{O}_5$  ( $M = \text{Mg}, \text{Ca}, \text{Zn}$ ). The subscripts under  $M$  ( $\text{Zn}, \text{Mg}, \text{Ca}$ ) specify the amount of intercalation considered for these values.

Formula	Voltage (V)	Capacity (mAh/g)
	<i>Mn<sub>2</sub>O<sub>4</sub></i>	
<b>Mg<sub>0-0.5</sub>Mn<sub>2</sub>O<sub>4</sub></b>	2.86	270
<b>Ca<sub>0-0.5</sub>Mn<sub>2</sub>O<sub>4</sub></b>	3.15	251
<b>Zn<sub>0-0.5</sub>Mn<sub>2</sub>O<sub>4</sub></b>	1.60	224
	<i>NiO<sub>2</sub></i>	
<b>Mg<sub>0-0.25</sub>NiO<sub>2</sub></b>	3.50	138
<b>Ca<sub>0-0.25</sub>NiO<sub>2</sub></b>	4.02	133

<b>Zn<sub>0-0.25</sub>NiO<sub>2</sub></b>	1.98	125
	<b>FePO<sub>4</sub></b>	
<b>Mg<sub>0-0.5</sub>FePO<sub>4</sub></b>	3.24	164
<b>Ca<sub>0-0.5</sub>FePO<sub>4</sub></b>	4.10	157
<b>Zn<sub>0-0.5</sub>FePO<sub>4</sub></b>	1.68	146
	<b><math>\delta</math>-V<sub>2</sub>O<sub>5</sub></b>	
<b>Mg<sub>0-1</sub>V<sub>2</sub>O<sub>5</sub></b>	2.56	260
<b>Ca<sub>0-1</sub>V<sub>2</sub>O<sub>5</sub></b>	3.04	242
<b>Zn<sub>0-1</sub>V<sub>2</sub>O<sub>5</sub></b>	1.11	217

The average voltage data calculated is given by:

$$V = - \frac{G[M_{x_2}Host] - G[M_{x_1}Host] - (x_2 - x_1)G[M]}{x_2 - x_1} \quad (1)$$

where  $G$  the Gibbs free energy of the compound is approximated by the DFT total energy at 0 K, and  $x_1$  and  $x_2$  are the fractions of MV cation intercalated respectively.

The specific capacity of the intercalation compound is given by:

$$Capacity = \frac{2V(x_2 - x_1)}{m[M_{x_2}Host] + m[M_{x_1}Host]} \quad (2)$$

where  $V$  is the average voltage (from Eq. 1) and  $m$  is the mass of the compound.

## References

1. D. Sheppard, R. Terrell, and G. Henkelman, Optimization methods for finding minimum energy paths. *J. Chem. Phys.*, 2008, **128**, 134106.
2. J. P. Perdew, K. Burke, and M. Ernzerhof, Generalized gradient approximation made simple, *Phys. Rev. Lett.*, 1996, **77**, 3865–3868.
3. M. Liu, Z. Rong, R. Malik, P. Canepa, A. Jain, G. Ceder, and K. A. Persson, Spinel compounds as multivalent battery cathodes: a systematic evaluation based on ab initio calculations, *Energy Environ. Sci.*, 2014, **8**, 964–974.
4. G. Kresse and D. Joubert, From ultrasoft pseudopotentials to the projector augmented-wave method, *Phys. Rev. B*, 1999, **59**, 1758–1775.
5. J. Carrasco, Role of van der Waals Forces in Thermodynamics and Kinetics of Layered Transition Metal Oxide Electrodes: Alkali and Alkaline-Earth Ion Insertion into V<sub>2</sub>O<sub>5</sub>, *J. Phys. Chem. C*, 2014, **118**, 19599–19607.
6. B. Zhou, H. Shi, R. Cao, X. Zhang, and Z. Jiang, Theoretical study on the initial stage of a magnesium battery based on a V<sub>2</sub>O<sub>5</sub> cathode, *Phys. Chem. Chem. Phys.*,

2014, **16**, 18578.